Surface Craft Motion Parameter Estimation Using Multipath Delay Measurements from Hydrophones

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***Abstract*— An equation-error (EE) method is described for estimating the three motion parameters of a watercraft (sound source) moving on the sea surface at a constant speed in a constant direction as it transits past a hydrophone located above the sea floor in a shallow water environment, using multipath delay measurements from the hydrophone. The motion parameter estimates are obtained via a linear least-squares (LLS) minimization followed by an algebraic transformation. A weighting matrix is derived for the LLS minimization to reduce the error variances of the motion parameter estimates. Computer simulation results show that the error variances approach the Cramer-Rao lower bounds for small multipath delay measurement errors. The effectiveness of the EE method is demonstrated using real hydrophone data. A method is then described for estimating the linear trajectory of the transiting source using the motion parameter estimates from the individual hydrophones of an underwater acoustic sensor network.**

I. INTRODUCTION

Passive sonar systems can be used to detect and localize fast surface craft that generate intense continuous broad-band sound underwater as a result of propeller cavitation [1]. In a shallow water environment, the signal emitted by a surface acoustic source arrives at a hydrophone located above the sea floor via a direct path and multipaths. Given the sensor height and water depth, the instantaneous range of the surface acoustic source from the sensor can be estimated using the difference in time of arrival (or multipath delay) measurement between the direct path and bottom-reflected path signals [1]-[3]. Also, if the source moves at a constant speed in a constant direction as it transits past the sensor, then the three source motion parameters (speed together with time and range at which the source is at the closest point of approach (CPA) to the sensor) can be estimated by observing the multipath delay over a sufficiently long time period that covers both inbound and outbound legs of the source transit. A common approach is to fit a multipath delay model, which is a nonlinear function of the three motion parameters, to the sequence of multipath delay measurements in a least-squares sense [1]. The three motion parameter values that minimize the squared deviations of the multipath delay observations from their predicted values provide the nonlinear least-squares (NLS) estimates of the speed, CPA time and CPA range of the source. The minimization is performed using numerical optimization methods which are not only computationally intensive but also require good initial estimates of the three motion parameters for fast convergence to the NLS solution.

An alternative approach to the problem is the equation-error (EE) method, which has been used previously to estimate the depth, CPA time and CPA range of an underwater acoustic source, assuming the source speed was known [4]. The main advantages of this approach are that closed-form solutions exist (thus requiring no initial estimates) and it is computationally efficient. In this paper, an EE method is proposed to estimate the speed, CPA time and CPA range of a surface craft (sound source) using multipath delay measurements from a single hydrophone. The method consists of a linear least-squares (LLS) minimization where a three-dimensional state vector, whose elements are simple algebraic functions of the speed, CPA time and CPA range of the source, is estimated. An algebraic transformation then converts the state vector estimate into the three motion parameter estimates. In order to reduce the error variances of the motion parameter estimates, a weighting matrix is derived for the LLS minimization. The error variances obtained from computer simulations of the proposed method with, and without, weighting are compared with the Cramer-Rao lower bounds (CRLBs). The proposed method is applied to real hydrophone data and typical results are presented to illustrate its effectiveness. A method is then described for estimating the linear trajectory of the transiting source using the state vector or motion parameter estimates from the individual hydrophones of an underwater acoustic sensor network.

II. MOTION PARAMETER ESTIMATION – SINGLE SENSOR

A. Problem Formulation

Figure 1 shows the geometry of a hydrophone located at a depth of d below the sea surface and a moving source which emits continuous broadband sound underwater as it transits past the sensor along a linear trajectory at a constant speed v on the sea surface. The sea floor is assumed to be *locally* flat around the sensor so that the sea-bottom-reflections of the sound emitted by the source to the sensor during the passage of the source past the sensor can be considered as if they originated from a mirror image source moving at a *constant* depth equal to twice the local water depth d_w. The source and the sensor are located at a height of h_t and h_r above the local sea bed respectively. The local source height h_t equals d_w and the local sensor height h_r equals $d_w - d$. At time τ_c , the source is at CPA to the sensor and its horizontal range from

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14. ABSTRACT
An equation-error (EE) method is described for estimating the three motion parameters of a watercraft (sound source) moving on the sea surface at a constant speed in a constant direction as it transits past a hydrophone located above the sea floor in a shallow water environment, using multipath delay measurements from the hydrophone. The motion parameter estimates are obtained via a linear least-squares (LLS) minimization followed by an algebraic transformation. A weighting matrix is derived for the LLS minimization to reduce the error variances of the motion parameter estimates. Computer simulation results show that the error variances approach the Cramer-Rao lower bounds for small multipath delay measurement errors. The effectiveness of the EE method is demonstrated using real hydrophone data. A method is then described for estimating the linear trajectory of the transiting source using the motion parameter estimates from the individual hydrophones of an underwater acoustic sensor network.

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the sensor is d_c. The slant range of the source from the sensor at time t is given by

$$R(t)=[v^2(t-\tau_c)^2+R_c^2]^{1/2} \tag{1}$$

where $R_c=[(h_t-h_r)^2+d_c^2]^{1/2}$ is the source's slant range at CPA. Squaring and then expanding both sides of (1) gives

$$R^2(t)=\mathbf{h}^T(t)\mathbf{x} \tag{2}$$

where $\mathbf{h}(t)=[t^2,-2t,1]^T$, $\mathbf{x}=[v^2,\tau_c v^2,\tau_c^2 v^2+R_c^2]^T$ and the superscript T denotes matrix or vector transpose.

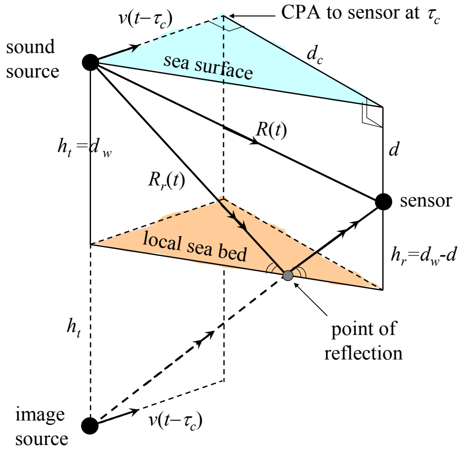

Fig.1. Geometry of a sensor located above a locally flat sea floor and passage of a surface sound source past this sensor. The image source is located at a depth equal to twice the local water depth or local source height.

Denote the difference in the times of arrival of the direct path and bottom-reflected path signals at the sensor (simply referred to as the multipath delay) as $\tau(t)$:

$$\tau(t)=[R_r(t)-R(t)]/c \tag{3}$$

where $R_r(t)=[R^2(t)+4h_t h_r]^{1/2}$ is the length of the bottom-reflected path and c is the speed of sound propagation in water. It can be shown using (3) that

$$R(t)=\frac{4h_t h_r-c^2\tau^2(t)}{2c\tau(t)}. \tag{4}$$

Let $\hat{\tau}(t)$ be an estimate of the multipath delay $\tau(t)$ at time t. Given the sensor height h_r, source height h_t and sound speed in water c, an estimate of $R^2(t)$ at time t, denoted as $y(t)\equiv\hat{R}^2(t)$, can be obtained by substituting $\hat{\tau}(t)$ for $\tau(t)$ in (4) and then squaring the result. Suppose there is an error of $\Delta\tau(t)$ in $\hat{\tau}(t)$, which results in an error of $e(t)\equiv\Delta R^2(t)$ in $y(t)$. Using (2), the observation $y(t)$ at time t_k can be written as

$$y(t_k)=\mathbf{h}^T(t_k)\mathbf{x}+e(t_k)\,,\quad 1\le k\le K \tag{5}$$

where K is the number of observations. The observation time period $t_1\le t\le t_k$ covers both inbound and outbound legs of the source transit. Equation (5) can be expressed in matrix form as

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{e} \tag{6}$$

where $\mathbf{y}=[y(t_1),\ldots,y(t_K)]^T$ is the observation vector, $\mathbf{e}=[e(t_1),\ldots,e(t_K)]^T$ is the observation error (or equation error) vector, $\mathbf{H}=[\mathbf{h}(t_1),\ldots,\mathbf{h}(t_K)]^T$ is the system matrix, and $\mathbf{x}=[x_1,x_2,x_3]^T$ is the (constant) state vector. The elements of $\mathbf{x}$ are simple algebraic functions of the three motion parameters v,τ_c and R_c, and vice versa:

$$x_1=v^2\,,\ x_2=\tau_c v^2\,,\ x_3=\tau_c^2 v^2+R_c^2 \tag{7}$$

$$v=x_1^{1/2}\,,\ \tau_c=x_1^{-1}x_2\,,\ R_c=(x_3-x_1^{-1}x_2^2)^{1/2}\,. \tag{8}$$

The objective is to estimate $\mathbf{x}$ for a given $\mathbf{y}$.

B. Algorithm

A LLS estimate of the state vector $\mathbf{x}$ is obtained by finding the vector $\hat{\mathbf{x}}$ that minimizes the following cost function:

$$J=(\mathbf{y}-\mathbf{H}\mathbf{x})^T\mathbf{W}(\mathbf{y}-\mathbf{H}\mathbf{x}) \tag{9}$$

where $\mathbf{W}$ is a positive definite, symmetric weighting matrix. The LLS estimate $\hat{\mathbf{x}}$ is given by

$$\hat{\mathbf{x}}=(\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}\mathbf{y}\,. \tag{10}$$

Once $\hat{\mathbf{x}}$ is computed, the EE estimates of the three motion parameters, denoted as $\hat{v},\hat{\tau}_c$ and $\hat{R}_c$, are readily obtained by substituting $\hat{\mathbf{x}}$ for $\mathbf{x}$ in (8).

If $\mathbf{W}$ is an identity matrix, then $\hat{\mathbf{x}}$ is an *unweighted* LLS estimate. Using a proper weighting matrix can reduce the error variances in $\hat{\mathbf{x}}$. If the observation errors $e(t_k)$ can be modelled as zero-mean random variables with a covariance matrix $\mathbf{R}_e$, a proper weighting matrix will be the inverse of $\mathbf{R}_e$. For small multipath delay measurement errors $\Delta\tau(t_k)$, the observation error $e(t_k)\equiv\Delta R^2(t_k)$ can be approximated to the first order as

$$e(t_k)\cong 2R(t_k)(dR/d\tau)\Delta\tau(t_k) \tag{11}$$

where the derivative $dR/d\tau$ can be computed using (4) as

$$dR/d\tau=-cR_r(t_k)/[R_r(t_k)-R(t_k)]\,. \tag{12}$$

Substituting (12) into (11) gives

$$e(t_k)\cong -2c[R^{-1}(t_k)-R_r^{-1}(t_k)]^{-1}\Delta\tau(t_k)\,. \tag{13}$$

Assuming that all multipath delay measurement errors $\Delta\tau(t_k)$, $1\le k\le K$, are independent, zero-mean, with a common variance of σ_τ^2, then the observation errors $e(t_k)$ are also independent, (approximately) zero-mean, and the covariance matrix $\mathbf{R}_e$ is a diagonal matrix whose kkth element, which equals the variance of $e(t_k)$, is given by

$$R_e(k,k)\cong 4c^2[R^{-1}(t_k)-R_r^{-1}(t_k)]^{-2}\sigma_\tau^2 \quad \text{for } 1\le k\le K\,. \tag{14}$$

With $\mathbf{W}=\mathbf{R}_e^{-1}$, it can be shown using (6) and (10) that the mean-square-error (MSE) of $\hat{\mathbf{x}}$, defined as the ensemble average of $(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^T$, is equal to $(\mathbf{H}^T\mathbf{R}_e^{-1}\mathbf{H})^{-1}$. In practice, $R_e(k,k)$ is computed using (14) with $R(t_k)$ and $R_r(t_k)$ replaced by their respective estimated values $y^{1/2}(t_k)$ and

$[y(t_k)+4h_t h_r]^{1/2}$. Also, the constant $4c^2\sigma_\tau^2$ in (14) can be ignored as it has no effect on $\hat{\mathbf{x}}$.

C. Computer Simulations

In the first scenario, the sensor height $h_r = 1$ m, source height (equal to water depth) $h_t = 20$ m, speed of sound in water $c = 1520$ m/s, source speed $v = 10$ m/s, CPA time $\tau_c = 0$ s, and CPA slant range $R_c = 50$ m. The multipath delay $\tau(t)$ was computed using (1) and (3) every 0.5 s over the time interval of $-10 \le t \le 10$ s. Multipath delay measurements were then generated by adding independent zero-mean Gaussian noise with a standard deviation of σ_τ to $\tau(t)$. For a given value of σ_τ, 1,000 simulation runs were performed, first with $\mathbf{W} = \mathbf{R}_e^{-1}$ (weighted EE) and then with $\mathbf{W}$ equal to an identity matrix (unweighted EE), and the bias errors, standard deviations (STDs) and root-mean-square errors (RMSEs) of $\hat{v}$, $\hat{\tau}_c$ and $\hat{R}_c$ were computed. Table 1 shows the simulation results obtained with, and without, weighting for (a) σ_τ = 10 µs and (b) σ_τ = 50 µs. Also included in Table 1 are the results obtained using the single-sensor NLS method [1] and the CRLBs. Both the EE and NLS methods were implemented in MATLAB®. The NLS method used the EE estimates to initialize the numerical (iterative) minimization. For the number of multipath delay measurements taken (41 in each simulation run), the computation time for the NLS method was about 100 times longer than that for the EE method.

TABLE 1. SIMULATION RESULTS FOR EE AND NLS METHODS USING SINGLE SENSOR, TOGETHER WITH CRLBS ON STDS, FOR TWO DIFFERENT MULTIPATH DELAY MEASUREMENT ERRORS: (A) σ_τ = 10 µS AND (B) σ_τ = 50 µS.

(a) σ_τ = 10µs		Unweighted EE	Weighted EE	NLS	CRLB
v (m/s)	bias	0.030	-0.070	0.004	
	STD	0.178	0.115	0.114	0.114
	RMSE	0.181	0.135	0.114	
τ_c (s)	bias	-0.001	0.000	0.001	
	STD	0.094	0.053	0.053	0.052
	RMSE	0.094	0.053	0.053	
R_c (m)	bias	-0.048	-0.050	0.001	
	STD	0.725	0.341	0.340	0.329
	RMSE	0.727	0.344	0.340	
(b) σ_τ = 50µs		**Unweighted EE**	**Weighted EE**	**NLS**	**CRLB**
v (m/s)	bias	0.910	-1.276	0.017	
	STD	1.245	0.617	0.607	0.569
	RMSE	1.542	1.417	0.607	
τ_c (s)	bias	-0.008	-0.001	0.001	
	STD	0.536	0.342	0.269	0.261
	RMSE	0.536	0.342	0.269	
R_c (m)	bias	-1.916	-1.455	0.001	
	STD	7.762	1.653	1.648	1.645
	RMSE	7.994	2.202	1.648	

Simulations have also been performed for a second scenario where R_c was decreased to 25 m while other conditions remained unchanged. Similar trends to Table 1 have been observed. The computer simulation results show that the NLS estimates are better than the EE estimates; not only their bias errors are smaller but their STDs are also closer to the CRLBs. This is not surprising as the multipath delay measurements contained additive independent zero-mean Gaussian noise and so the NLS estimates are also maximum likelihood estimates. The STDs of the weighted EE estimates approach the CRLBs as the value of σ_τ decreases, and their accuracy is better than that of the unweighted EE estimates, especially for R_c when the value of σ_τ is large.

D. Experimental Results

In a shallow water experiment (water depth ≈ 20 m), eight hydrophones were located at a nominal height of 1 m above the sea floor. The sensor configuration was (almost) linear with an intersensor spacing of 14 m. A small surface vessel travelled in a straight line at a nominal speed of 9 knots (4.63 m/s) past the hydrophone array at a horizontal distance of less than 90 m from the centre of the array. The vessel trajectory was apparently perpendicular to the array axis. The output of each hydrophone was sampled at 250 kHz. The data recorded from each hydrophone were processed in non-overlapped blocks, each containing 131,072 samples (about 0.52 s of data). Each data block was subdivided into five 75% overlapped blocks, each consisting of 65,536 samples. The power spectra of these five subdivided data blocks were computed using the Fast Fourier Transform (FFT) and then averaged to produce a smoothed power spectrum. An inverse FFT was applied to the logarithm of the smoothed power spectrum to produce a cepstrum. The time lag at which the cepstrum attains its maximum value provides an estimate of the multipath delay. In this way, a sequence of multipath delay estimates was obtained for each hydrophone.

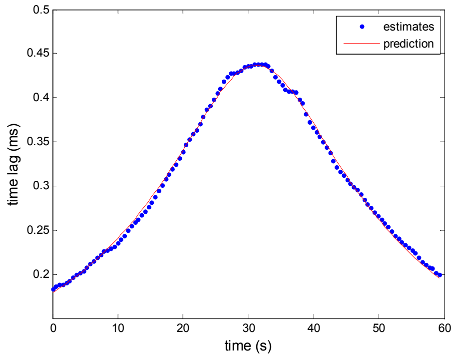

Fig. 2. Experimental results showing the sequence of multipath delay estimates from sensor 6 and the prediction computed using the estimated motion parameter values.

The proposed (weighted) EE method was applied in turn to the sequence of multipath delay estimates from each sensor. Figure 2 shows (as dots) the sequence of multipath delay estimates from sensor 6 and (as a solid line) the prediction computed using the estimated source motion parameter values from that sensor. Figure 3 shows (as dots) the EE estimates of the speed, CPA time and CPA slant range of the surface vessel from each of the eight sensors. Also included (as circles) in Fig. 3 are the three motion parameter estimates obtained using the single-sensor NLS method for comparison purposes. The two sets of estimates are in good agreement. The speed estimates from both methods also agree closely with the nominal speed. Note that since the vessel trajectory is apparently perpendicular (possibly at a small inclination angle) to the array axis, its CPA horizontal ranges to any two adjacent sensors should differ by a value equal approximately to the intersensor spacing, i.e., 14 m. The CPA slant range estimates obtained by both methods were converted into CPA horizontal range estimates and then the difference in the estimates for every two adjacent sensors was computed. For the EE method, the mean and standard deviation in these differences are 14.11 and 1.02 m respectively, while for the NLS method, they are 14.10 and 1.01 m respectively.

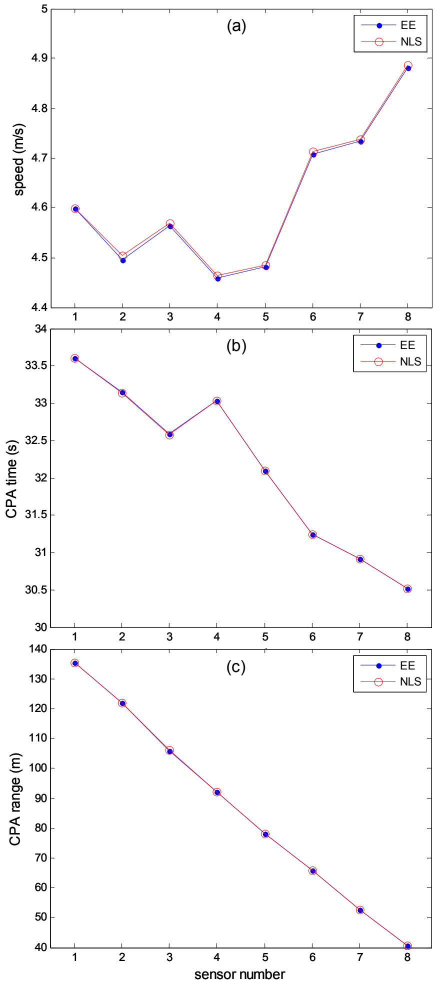

Fig. 3. Experimental results showing both EE and NLS estimates of (a) speed, (b) CPA time and (c) CPA range of the surface vessel.

III. Motion Parameter Estimation – Multiple Sensors

A. Source-Sensor Model

Figure 4 shows the general geometrical configuration for a network of N widely distributed underwater acoustic sensors and a moving source which emits continuous broadband sound underwater as it transits over the sensor network along a linear trajectory at a constant velocity V on the sea surface. The XY-plane coincides with the planar sea surface. The position of sensor n is given by (X_n, Y_n, d_n), where d_n is the depth of sensor n, for $1 \le n \le N$. It is assumed that sensor 1 is located directly below the origin so that $X_1 = Y_1 = 0$. The trajectory of the source on the XY-plane is described by

$$\begin{aligned} X(t) &= d_c \cos\theta_c + (t - \tau_c) V \sin\theta_c \\ Y(t) &= d_c \sin\theta_c - (t - \tau_c) V \cos\theta_c \end{aligned} \tag{15}$$

where τ_c is the time when the source is at CPA to sensor 1; d_c and θ_c are the respective horizontal range and azimuth angle of the source at CPA to sensor 1. The source trajectory is specified by the four motion parameters: V, τ_c, d_c and θ_c. Note that V can be negative; the sign convention is that V is positive (negative) when sensor 1 is on the right (left) hand side of the source as the source moves along its trajectory.

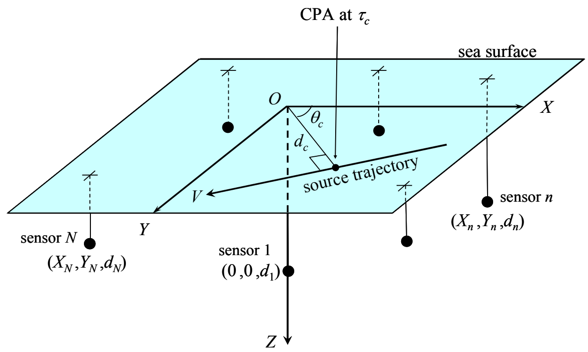

Fig. 4. General geometrical configuration for a network of N underwater acoustic sensors and transit of a surface sound source over the sensor network.

The sea floor is assumed to be *locally* flat around each sensor – see Fig. 1. At time $\tau_{c,n}$, the source is at CPA to sensor n and its horizontal and slant ranges from this sensor are $d_{c,n}$ and $R_{c,n}$ respectively, for $1 \le n \le N$, with $\tau_{c,1} \equiv \tau_c$, $d_{c,1} \equiv d_c$ and $R_{c,n} \equiv R_c$. The source and sensor n are located at a height of $h_{t,n}$ and $h_{r,n}$ above the local sea bed respectively, where $h_{t,n}$ equals the local water depth $d_{w,n}$ and $h_{r,n} = d_{w,n} - d_n$.

B. Algorithm

The EE method described in the previous section can be applied to the sequence of multipath delay measurements from each sensor to estimate the state vector and hence the speed as well as the time and slant range at which the source is at CPA to that sensor. The sequence of multipath delay measurements from each sensor should be taken during the passage of the source past that sensor. Thus the measurement times (i.e., instants at which the multipath delay measurements are taken) and number of measurements may vary from sensor to sensor. Only the source speed $|V|$ can be estimated with a single sensor. Let $v = |V|$ and denote the state vector for sensor n as $\mathbf{x}_n = [x_{1,n}, x_{2,n}, x_{3,n}]^T$. Parallel to (7) and (8), $x_{1,n}$, $x_{2,n}$ and $x_{3,n}$ are related to v, $\tau_{c,n}$ and $R_{c,n}$ via

$$x_{1,n} = v^2, \; x_{2,n} = \tau_{c,n} v^2, \; x_{3,n} = \tau_{c,n}^2 v^2 + R_{c,n}^2 \tag{16}$$

$$v = x_{1,n}^{1/2}, \; \tau_{c,n} = x_{1,n}^{-1} x_{2,n}, \; R_{c,n} = (x_{3,n} - x_{1,n}^{-1} x_{2,n}^2)^{1/2}. \tag{17}$$

Since $R_{c,n}^2 = d_{c,n}^2 + (h_{t,n} - h_{r,n})^2$, it follows by substituting the expression for $R_{c,n}$ in (17) into this expression that

$$d_{c,n} = [x_{3,n} - x_{1,n}^{-1} x_{2,n}^2 - (h_{t,n} - h_{r,n})^2]^{1/2}. \tag{18}$$

Denote the (weighted) LLS estimate of the state vector $\mathbf{x}_n$ from sensor n as $\hat{\mathbf{x}}_n = [\hat{x}_{1,n}, \hat{x}_{2,n}, \hat{x}_{3,n}]^T$ and the corresponding EE estimates of the source speed v, CPA time $\tau_{c,n}$ and CPA horizontal range $d_{c,n}$ as $\hat{v}_n$, $\hat{\tau}_{c,n}$ and $\hat{d}_{c,n}$ respectively, for $1 \le n \le N$. These EE estimates are computed by substituting $\hat{\mathbf{x}}_n$ for $\mathbf{x}_n$ in (17) and (18). An improved estimate of v is obtained by averaging the source speed estimates from the individual sensors, i.e., $\hat{v} = N^{-1}\sum_{n=1}^{N}\hat{v}_n$. The estimates of the CPA time τ_c, denoted as $\hat{\tau}_c$, and CPA horizontal range d_c, denoted as $\hat{d}_c$, are simply given by $\hat{\tau}_{c,1}$ and $\hat{d}_{c,1}$ respectively.

An EE approach is adopted to obtain an estimate of the CPA azimuth angle θ_c using the state vector or source motion parameter estimates from the N sensors. Consider the source trajectory and the projection of the sensors onto the XY-plane as shown in Fig. 5, where sensor 1 is located at the origin and sensor n at (X_n, Y_n). The unit directional vector $\mathbf{v}$ is defined as $\mathbf{v} = [\sin\theta_c, -\cos\theta_c]^T$ so that the source's travel direction is given by $\mathrm{sgn}(V)\mathbf{v}$, where $\mathrm{sgn}(V)$ is the sign of V. The unit directional vector $\mathbf{u} = [\cos\theta_c, \sin\theta_c]^T$ is obtained by rotating $\mathbf{v}$ by 90° in the anti-clockwise direction. From Fig. 5, it can be shown for $2 \le n \le N$ that

$$d_{c,1} - d_{c,n} = \mathbf{r}_n^T\mathbf{u} \tag{19}$$

$$V(\tau_{c,n} - \tau_{c,1}) = \mathbf{r}_n^T\mathbf{v} \tag{20}$$

where $\mathbf{r}_n = [X_n, Y_n]^T$ is the position vector of sensor n.

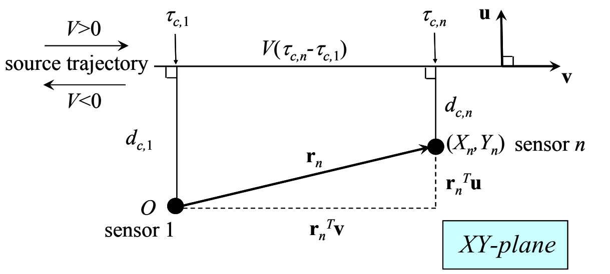

Fig. 5. Source trajectory and projection of sensors onto the XY-plane.

Substituting $V = \mathrm{sgn}(V)v$ into (20) gives

$$v(\tau_{c,n} - \tau_{c,1}) = \mathbf{r}_n^T\boldsymbol{\alpha} \tag{21}$$

where the unit vector $\boldsymbol{\alpha} = \mathrm{sgn}(V)\mathbf{v}$ indicates the source's travel direction. Using (17), the left hand side of (21) can be expressed as

$$v(\tau_{c,n} - \tau_{c,1}) = f(\mathbf{x}_n) - f(\mathbf{x}_1) \tag{22}$$

where $f(\mathbf{x}) = x_1^{-1/2}x_2$. Replacing $\{v, \tau_{c,n}\}$ and $\{v, \tau_{c,1}\}$ by their estimates: $\{\hat{v}_n, \hat{\tau}_{c,n}\}$ and $\{\hat{v}_1, \hat{\tau}_{c,1}\}$ modifies (21) to

$$\hat{v}_n\hat{\tau}_{c,n} - \hat{v}_1\hat{\tau}_{c,1} = \mathbf{r}_n^T\boldsymbol{\alpha} + w_{n-1}, \quad 2 \le n \le N \tag{23}$$

where w_{n-1} is the error in observing $v(\tau_{c,n} - \tau_{c,1})$ (or equation error). Equation (23) can be written in matrix form:

$$\mathbf{z} = \mathbf{S}\boldsymbol{\alpha} + \mathbf{w} \tag{24}$$

where the vectors $\mathbf{z} = [\hat{v}_2\hat{\tau}_{c,2} - \hat{v}_1\hat{\tau}_{c,1}, \ldots, \hat{v}_N\hat{\tau}_{c,N} - \hat{v}_1\hat{\tau}_{c,1}]^T$ and $\mathbf{w} = [w_1, \ldots, w_{N-1}]^T$, and the matrix $\mathbf{S} = [\mathbf{r}_2, \ldots, \mathbf{r}_N]^T$. The LLS estimate of $\boldsymbol{\alpha}$ is given by

$$\hat{\boldsymbol{\alpha}} = (\mathbf{S}^T\mathbf{W}_u\mathbf{S})^{-1}\mathbf{S}^T\mathbf{W}_u\mathbf{z} \tag{25}$$

where $\mathbf{W}_u$ is a weighting matrix. Once $\hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \hat{\alpha}_2]^T$ is computed, the unit vector $\mathbf{u}$ can be estimated using the relations between $\mathbf{u}$ and $\mathbf{v}$ and between $\mathbf{v}$ and $\boldsymbol{\alpha}$ as $\hat{\mathbf{u}} = [\hat{u}_1, \hat{u}_2]^T$, where $\hat{u}_1 = -\mathrm{sgn}(V)\hat{\alpha}_2$ and $\hat{u}_2 = \mathrm{sgn}(V)\hat{\alpha}_1$, provided that the sign of V is known. The estimate of the CPA azimuth angle, denoted as $\hat{\theta}_c$, is then given by $\hat{\theta}_c = \tan^{-1}(\hat{u}_2/\hat{u}_1)$.

If the observation errors w_{n-1} can be modelled as zero-mean random variables with a covariance matrix $\mathbf{R}_w$, a proper weighting matrix for (25) will be the inverse of $\mathbf{R}_w$. The observation error w_{n-1} in (23) is caused by the errors in the state vector estimates $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_n$. Using (22), it can be approximated (to the first order) as

$$w_{n-1} \cong \nabla^T f(\hat{\mathbf{x}}_n)\Delta\mathbf{x}_n - \nabla^T f(\hat{\mathbf{x}}_1)\Delta\mathbf{x}_1 \tag{26}$$

where $\nabla f(\hat{\mathbf{x}}_k)$ is the gradient of $f(\mathbf{x})$ evaluated at $\hat{\mathbf{x}}_k$, and $\Delta\mathbf{x}_k$ is the error in $\hat{\mathbf{x}}_k$, for $1 \le k \le N$. By definition, $\Delta\mathbf{x}_k = \hat{\mathbf{x}}_k - \mathbf{x}_k$. For small multipath delay measurement errors, a first-order approximation gives $E[\Delta\mathbf{x}_k] \approx 0$ and hence $E[w_{n-1}] \approx 0$. Therefore, $\mathbf{R}_w \approx E[\mathbf{w}\mathbf{w}^T]$. Assuming that $\Delta\mathbf{x}_k$ and $\Delta\mathbf{x}_l$ are independent for $k \ne l$ and $1 \le k, l \le N$, it can be shown using (26) that

$$E[w_{m-1}w_{n-1}] \cong \nabla^T f(\hat{\mathbf{x}}_1)E[\Delta\mathbf{x}_1\Delta\mathbf{x}_1^T]\nabla f(\hat{\mathbf{x}}_1) \tag{27}$$

$$E[w_{n-1}^2] \cong E[w_{m-1}w_{n-1}] + \nabla^T f(\hat{\mathbf{x}}_n)E[\Delta\mathbf{x}_n\Delta\mathbf{x}_n^T]\nabla f(\hat{\mathbf{x}}_n) \tag{28}$$

for $m \ne n$ and $2 \le m, n \le N$. In (27) and (28), $E[\Delta\mathbf{x}_k\Delta\mathbf{x}_k^T]$ is the MSE of $\hat{\mathbf{x}}_k$, which is equal to $(\mathbf{H}_k^T\mathbf{R}_{e,k}^{-1}\mathbf{H}_k)^{-1}$, where $\mathbf{H}_k$ and $\mathbf{R}_{e,k}$ are the system matrix and covariance matrix of the observation error vector $\mathbf{e}$ for sensor k, respectively.

The sign of the source velocity V is yet to be determined. Substituting $\hat{d}_{c,1}$, $\hat{d}_{c,n}$ and $\hat{\mathbf{u}}$ for $d_{c,1}$, $d_{c,n}$ and $\mathbf{u}$ in (19) results in an equation error given by

$$\varepsilon_{n-1}(V) = \hat{d}_{c,1} - \hat{d}_{c,n} - \mathrm{sgn}(V)\mathbf{r}_n^T[-\hat{\alpha}_2, \hat{\alpha}_1]^T. \tag{29}$$

Expressing (29) for $2 \le n \le N$ in matrix form gives

$$\boldsymbol{\varepsilon}(V) = \mathbf{b} - \mathrm{sgn}(V)\mathbf{S}[-\hat{\alpha}_2, \hat{\alpha}_1]^T \tag{30}$$

where the vectors $\mathbf{b} = [\hat{d}_{c,1} - \hat{d}_{c,2}, \ldots, \hat{d}_{c,1} - \hat{d}_{c,N}]^T$ and $\boldsymbol{\varepsilon}(V) = [\varepsilon_1(V), \ldots, \varepsilon_{N-1}(V)]^T$. Define the cost (or error) function $p(V) = \boldsymbol{\varepsilon}^T(V)\boldsymbol{\varepsilon}(V)$ and denote the estimate of V as $\hat{V}$. If $p(\hat{v}) > p(-\hat{v})$, then $\hat{V} = -\hat{v}$ $(\hat{V} < 0)$; otherwise $\hat{V} = \hat{v}$ $(\hat{V} > 0)$.

C. Computer Simulations

Five sensors were located at a depth of 19 m below the sea surface in a cross shaped configuration. The (X,Y)-coordinates of sensors 1 to 5 were (0,0), (-50,0), (50,0), (0,50) and (0,-50) m respectively. A surface acoustic source travelled through the network of sensors and the values of the source motion parameters were $V = 10$ m/s, $\tau_c = 0$ s, $d_c = 25$ m and $\theta_c = 90^\circ$. The sea floor was flat throughout the area where the sensors were located, and the water depth was 20 m. For each sensor, a noisy multipath delay measurement was generated every 0.5 s over a 20 s time interval centred at the time of CPA to that sensor, with the measurement errors being independent, zero-mean Gaussian distributed with a common variance of σ_τ. For a given value of σ_τ, 1,000 simulation runs were performed and the statistics including bias errors, STDs and RMSEs of $\hat{V}, \hat{\tau}_c, \hat{d}_c$ and $\hat{\theta}_c$ were compiled.

Figure 6 shows the series of multipath delay measurements from each sensor in a typical simulation run for $\sigma_\tau = 10$ µs. Table 2 shows the compiled statistics of the source motion parameter estimates for (a) σ_τ = 10 µs and (b) σ_τ = 50 µs. The RMSE of $\hat{\theta}_c$ increases with the value of σ_τ from 0.34° to 2°. Also included in Table 2 are the statistics obtained using the NLS method [5] and the CRLBs for comparison purposes. The NLS method used the source motion parameter estimates from the proposed method as the initial estimates. Though the NLS method provides more accurate estimates than the proposed method, the latter provides closed-form solutions and is computationally more efficient than the former. Computer simulations have also been performed for another scenario where $\theta_c = 45^\circ$ while other conditions remained unchanged. Similar results to Table 2 were obtained.

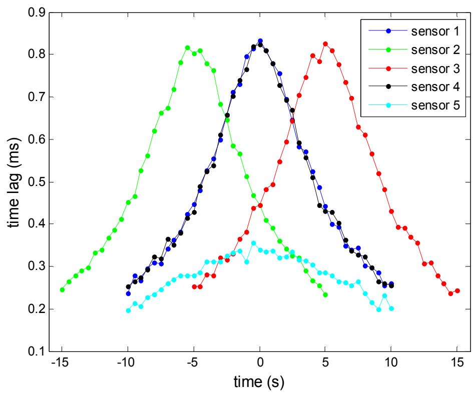

Fig. 6. Sequence of multipath delay measurements from each sensor in a typical simulation run for σ_τ = 10 µs.

IV. CONCLUSIONS

Multipath propagation can be utilized for localizing fast surface watercraft in a shallow water environment. An EE method has been described for estimating the speed, CPA time and CPA range of a sound source moving on the sea surface at a constant speed in a constant direction as it transits past a hydrophone located above the sea floor, using multipath delay measurements from the hydrophone. The performance of the EE method was studied by computer simulations. Using a proper weighting matrix reduces the error variances and improves the accuracy of the EE estimates. Experimental results were presented to demonstrate the effectiveness of the EE method. A method has also been described for estimating the four motion parameters (which specify the linear trajectory) of the transiting source using the EE estimates from the individual hydrophones of an underwater acoustic sensor network. Computer simulations produced encouraging results. The main advantages of the proposed methods over the NLS methods are that closed-form expressions exist for the motion parameter estimates and they are computationally efficient. The proposed methods can be used to provide initial estimates for the NLS methods. Future work will include studying the effects of the sensor configuration on the performance of the proposed method for a network of sensors and verifying the effectiveness of the method using real hydrophone data.

TABLE 2. SIMULATION RESULTS FOR EE AND NLS METHODS USING FIVE SENSORS, TOGETHER WITH CRLBS ON STDS, FOR TWO DIFFERENT MULTIPATH DELAY MEASUREMENT ERRORS: (A) σ_τ = 10 µS AND (B) σ_τ = 50 µS.

(a) σ_τ = 10µs		EE	NLS	CRLB
V (m/s)	bias	-0.052	-0.000	
	STD	0.053	0.023	0.023
	RMSE	0.075	0.023	
τ_c (s)	bias	0.001	-0.000	
	STD	0.025	0.013	0.013
	RMSE	0.025	0.013	
d_c (m)	bias	0.002	0.001	
	STD	0.207	0.091	0.091
	RMSE	0.207	0.091	
θ_c (rad)	bias	0.000	0.000	
	STD	0.006	0.002	0.002
	RMSE	0.006	0.002	
(b) σ_τ = 50µs		**EE**	**NLS**	**CRLB**
V (m/s)	bias	-1.090	0.009	
	STD	0.292	0.121	0.117
	RMSE	1.128	0.121	
τ_c (s)	bias	0.000	-0.003	
	STD	0.142	0.064	0.064
	RMSE	0.142	0.064	
d_c (m)	bias	-0.136	-0.008	
	STD	1.069	0.459	0.456
	RMSE	1.078	0.459	
θ_c (rad)	bias	-0.001	0.000	
	STD	0.035	0.012	0.012
	RMSE	0.035	0.012	

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